Int. J. Heat Mass Transfer. Vol. 25, No. 8, pp. 1247–1249, 1982
Printed in Great Britain

0017-9310/82/081247-03 $03.00/0

FILM BOILING ABOUT TWO-DIMENSIONAL AND AXISYMMETRIC ISOTHERMAL BODIES OF ARBITRARY SHAPE IN A POROUS MEDIUM

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(*Received* 23 *November* 1981 *and in final form* 18 *January* 1982)

NOMENCLATURE

A, area of the heating surface;
C, constant defined in equation (31);
c_p, specific heat of the fluid at constant pressure;
D, function defined in equations (10) and (13);
f, dimensionless stream function;
G, function defined in equations (11) and (14);
g, gravitational acceleration;
h, local heat transfer coefficient;
H, function defined in equations (10) and (13);
h_{fg}, latent heat of vaporization;
K, permeability of the porous medium;
k, thermal conductivity of the porous medium;
L, slant height of a wedge or a cone;
l, a characteristic length;
$\dot{m}_{\delta}$, evaporation rate at vapor–liquid interface;
n, constant defined in equations (1) and (2); $n = 0$ for 2-dim. bodies and $n = 1$ for axisymmetric bodies;
Nu_x, local Nusselt number;
p, pressure;
q, local heat transfer rate;
$\bar{q}$, average heat transfer rate;
R, property ratio of the vapor and the liquid phase, $\left[\dfrac{\rho_v}{\rho_\infty}\dfrac{\mu_L\alpha_v(\rho_\infty-\rho_v)C_{pL}}{\mu_v\alpha_L\rho_\infty\beta_L h_{fg}}\right]^{1/2}$;
r, radial distance from the axis to the surface of the axisymmetric bodies;
Ra, Rayleigh number;
Sc, dimensionless degree of subcooling of liquid, $C_{pL}(T_s - T_\infty)/h_{fg}$;
Sh, $Sh \equiv C_{pv}(T_w - T_s)/h_{fg}$, dimensionless degree of wall superheating;
T, temperature;
u, Darcy's velocity, x-direction;
v, Darcy's velocity, y-direction;
x, coordinate along the surface;
y, coordinate perpendicular to the surface.

Greek symbols

α, equivalent thermal diffusivity;
β, coefficient of thermal expansion;
δ, boundary layer thickness;
η, similarity variable;
θ, dimensionless temperature;
μ, viscosity of the fluid;
ρ, density of the fluid;
ϕ, angle measured from the downward vertical to the y-axis;
ψ, stream function;
χ, dimensionless distance in the x-direction.

Subscripts

s, saturated condition;
v, vapor phase;
L, liquid phase;
∞, condition at infinity;
w, condition at the wall.

INTRODUCTION

RECENT interest on the study of film boiling about heated isothermal bodies embedded in a permeable medium is motivated by its applications to geothermal energy utilization. For buoyancy-induced flow about a superheated body where temperature and pressure are not increasing or decreasing simultaneously along a streamline, Parmentier [1] has shown that the transition from the subcooled liquid to superheated steam is abrupt such that a distinct interface exists between the superheated vapor zone adjacent to the heated surface and a liquid water zone away from the surface. The absence of a 2-phase region results in considerable mathematical simplifications which lead to similarity solutions for film boiling about a vertical flat plate and a vertical cylinder in a porous medium at high Rayleigh numbers [2–4]. In this paper, the approach adopted previously by Cheng and coworkers [2, 3] is applied to the problem of film boiling about 2-dim. and axisymmetric isothermal bodies of arbitrary shape in a subcooled permeable medium. With a generalized similarity transformation similar to those used by Merkin [5] for problems of free convection in porous media, it is shown that the resulting ordinary differential equations and boundary conditions for the present generalized problem reduce to those of film boiling about a vertical flat plate [2]. Applications to film boiling about a wedge, a cone, a horizontal cylinder, and a sphere embedded in a subcooled permeable medium are discussed.

ANALYSIS

Consider the problem of film boiling about 2-dim. and axisymmetric bodies of arbitrary shape in a porous medium as shown in Fig. 1 where x is the coordinate measured along the heated surface from the lowest point, y is the coordinate perpendicular to the surface, and $\phi = \phi(x)$ is the angle between the coordinate y and the vertical direction. When the wall is maintained at a superheated temperature, it is assumed that a vapor film with thickness δ_v will form adjacent to the heated surface. On the basis of the 2-phase boundary layer theory, the governing equations for the porous medium filled with the superheated vapor at $y < \delta_v$ are [2]

$$\frac{1}{r^n}\frac{\partial\psi_v}{\partial y} = \frac{K}{\mu_v}(\rho_\infty - \rho_v)g\sin\phi \qquad (1)$$

$$\frac{1}{r^n}\left(\frac{\partial\psi_v}{\partial y}\frac{\partial T_v}{\partial x} - \frac{\partial\psi_v}{\partial x}\frac{\partial T_v}{\partial y}\right) = \alpha_v\frac{\partial^2 T_v}{\partial y^2} \qquad (2)$$

where the subscripts v and ∞ denote quantities associated with the vapor phase and at infinity; $n = 0$ for 2-dim. bodies and $n = 1$ for axisymmetric bodies; $r(x)$ is the radial distance from the axis to the surface of the axisymmetric bodies; g is

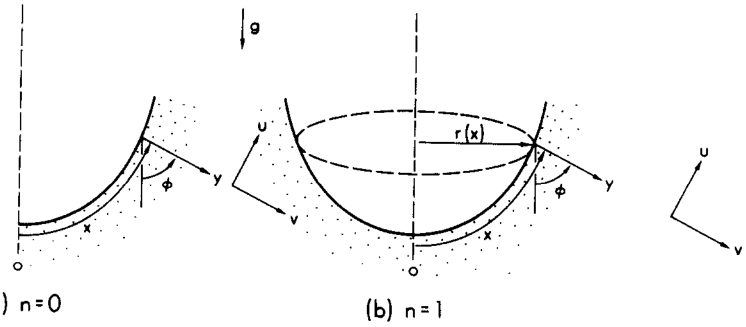

FIG. 1. Coordinate system for (a) a two-dimensional body and (b) an axisymmetric body.

the gravitational acceleration; ρ_v and μ_v are the density and viscosity of the vapor phase; K is the intrinsic permeability of the porous medium; α_v is the thermal diffusivity of the porous medium saturated with vapor; T_v is the temperature of the vapor and the porous medium; and ψ_v is the stream function of the vapor defined as

$$u_v = \frac{1}{r^n}\frac{\partial\psi_v}{\partial y}$$

and

$$v_v = -\frac{1}{r^n}\frac{\partial\psi_v}{\partial x}$$

with u_v and v_v denoting the Darcian velocities of the vapor in the x and y directions. The governing equations for the liquid phase at $y > \delta_v$ are [2]

$$\frac{1}{r^n}\frac{\partial\psi_L}{\partial y} = \frac{K\rho_\infty\beta g}{\mu_L}(T_L - T_\infty)\sin\phi \tag{3}$$

$$\frac{1}{r^n}\left(\frac{\partial\psi_L}{\partial y}\frac{\partial T_L}{\partial x} - \frac{\partial\psi_L}{\partial x}\frac{\partial T_L}{\partial y}\right) = \alpha_L\frac{\partial^2 T_L}{\partial y^2} \tag{4}$$

where the subscript L denotes the quantities associated with the liquid phase; β is the thermal expansion coefficient of the liquid phase; and ψ_L is the stream function of the liquid phase defined as

$$u_L = \frac{1}{r^n}\frac{\partial\psi_L}{\partial y}$$

and

$$v_L = -\frac{1}{r^n}\frac{\partial\psi_L}{\partial x}.$$

The boundary conditions at the wall and at infinity are

$$y = 0: \quad \frac{\partial\psi_v}{\partial x} = 0, \quad T_v = T_w \tag{5a, b}$$

and

$$y \to \infty: \quad \frac{\partial\psi_L}{\partial y} = 0, \quad T_L = T_\infty \tag{6a, b}$$

At the vapor–liquid interface ($y = \delta_v$), the continuity of temperature, mass flow and energy flux give [2]

$$T_v = T_s = T_L \tag{7}$$

$$\dot{m}_\delta = \frac{\rho_v}{r^n}\left[\frac{\partial\psi_v}{\partial y}\frac{d\delta_v}{dx} + \frac{\partial\psi_v}{\partial x}\right]_{y=\delta_v} = \frac{\rho_v}{r^n}\left[\frac{\partial\psi_L}{\partial y}\frac{d\delta_v}{dx} + \frac{\partial\psi_L}{\partial x}\right]_{y=\delta_v}, \tag{8}$$

$$-k_{m,v}\left(\frac{\partial T_v}{\partial y}\right)_{y=\delta_v} = \dot{m}_\delta h_{fg} - k_{m,L}\left(\frac{\partial T_L}{\partial y}\right)_{y=\delta_v} \tag{9}$$

where $k_{m,v}$ and $k_{m,L}$ are the equivalent thermal conductivities of the porous medium filled with the vapor and the liquid phases respectively; $\dot{m}_\delta$ is the evaporation rate and h_{fg} is the latent heat of vaporization of the liquid phase at the saturation temperature T_s.

We now introduce the following similarity transformations for the vapor phase:

$$\eta_v = Ra_v^{1/2}\left(\frac{y}{l}\right)\frac{H(\chi)D^n(\chi)}{G(\chi)} \tag{10}$$

$$\psi_v = \alpha_v Ra_v^{1/2}G(\chi)f_v(\eta_v)l^n \tag{11}$$

$$\theta_v(\eta_v) = \frac{T_v - T_s}{T_w - T_s} \tag{12}$$

and for the liquid phase

$$\eta_L = Ra_L^{1/2}\left(\frac{y-\delta_v}{l}\right)\frac{H(\chi)D^n(\chi)}{G(\chi)} \tag{13}$$

$$\psi_L = \alpha_v Ra_L^{1/2}G(\chi)f_L(\eta_L)l^n \tag{14}$$

$$\theta_L(\eta_L) = \frac{T_L - T_\infty}{T_s - T_\infty} \tag{15}$$

where $Ra_v = K(\rho_\infty - \rho_v)gl/\mu_v\alpha_v$ and $Ra_L = K\rho_\infty g\beta(T_s - T_\infty)l/\mu_L\alpha_L$ are the Rayleigh numbers of the vapor and the liquid phases with l denoting a characteristic length; $H(\chi) = \sin\phi(\chi)$, $D(\chi) = r(x)/l$, and

$$G(\chi) = \left[\int_0^\chi D^{2n}(t)H(t)\,dt\right]^{1/2}$$

where $\chi = x/l$. Substituting equations (10)–(15) into equations (1)–(9) yields the following ordinary differential equations:

$$f_v' = 1 \tag{16}$$

$$\theta_v'' + \tfrac{1}{2}f_v\theta_v' = 0 \tag{17}$$

and

$$f_L' = \theta_L \tag{18}$$

$$\theta_L'' + \tfrac{1}{2}f_L\theta_L' = 0 \tag{19}$$

subject to the boundary conditions

$$f_v(0) = \theta_v(0) - 1 = 0 \tag{20a, b}$$

$$f_L'(\infty) = \theta_L(\infty) = 0 \tag{21a, b}$$

and the interface conditions

$$\theta_v(\eta_{v\delta}) = 0, \quad \theta_L(0) = 1 \tag{22a, b}$$

$$f_L(0) = R\eta_{v\delta}/(Sc)^{1/2} \tag{23}$$

$$Sh = \left[\frac{Sc^{3/2}}{R}\theta_L'(0) - \frac{\eta_{v\delta}}{2}\right]\Big/\theta_v'(\eta_{v\delta}) \tag{24}$$

where $\eta_{v\delta} = Ra_v^{1/2}\delta_v/x$,

$$R = \frac{\rho_v}{\rho_\infty}\left[\frac{\mu_L\alpha_v(\rho_\infty - \rho_v)c_{pL}}{\mu_v\alpha_L\rho_\infty h_{fg}\beta}\right]^{1/2},$$

$Sc = c_{pL}(T_s - T_\infty)/h_{fg}$ and $Sh = c_{pv}(T_w - T_s)/h_{fg}$ with $c_{pv} = k_{m,v}/\rho_v\alpha_v$ and $c_{pL} = k_{m,L}/\rho_L\alpha_L$ denoting the specific heats of the porous medium that filled with the vapor and liquid phases respectively. Equations (16)–(19) with boundary conditions (20)–(24) have been integrated numerically by Cheng and Verma [2].

RESULTS AND DISCUSSION

In terms of the transformed variables, the Darcian velocities in the vapor and the liquid phases are

$$u_v = \left[\frac{K(\rho_\infty - \rho_v)g}{\mu_v}\right]H(\chi) \qquad (25)$$

$$v_v = -\frac{K(\rho_\infty - \rho_v)g}{\mu_v}\left(\frac{y}{l}\right)\frac{F'(\chi)G(\chi)}{D^n(\chi)} - \left[\frac{K\alpha_v(\rho_\infty - \rho_v)g}{\mu_v l}\right]^{1/2}\frac{D^n(\chi)H(\chi)}{G(\chi)}(\eta_v/2) \qquad (26)$$

and

$$u_L = \frac{K\rho_\infty\beta g(T_s - T_\infty)}{\mu_L}\theta_L(\eta_L)H(\chi) \qquad (27)$$

$$v_L = -\frac{K\rho_\infty\beta g(T_s - T_\infty)}{\mu_L}\left(\frac{y}{l}\right)\frac{F'(\chi)G(\chi)}{D^n(\chi)}\theta_L(\eta_L) - \left[\frac{K\alpha_L\rho_\infty\beta g(T_s - T_\infty)}{\mu_L l}\right]^{1/2}\frac{D^n(\chi)H(\chi)f_L(\eta_L)}{2G(\chi)} \qquad (28)$$

where $F(\chi) = D^n(\chi)H(\chi)/G(\chi)$ and $F'(\chi) = dF/d\chi$.

The local surface heat flux is given by

$$q_w = -k_{m,v}\left(\frac{\partial T}{\partial y}\right)_{y=0} = k_{m,v}\frac{Ra_v^{1/2}}{l}\frac{D^n(\chi)H(\chi)(T_w - T_s)}{G(\chi)}[-\theta_v'(0)] \qquad (29)$$

where the values of $-\theta_v'(0)$ as a function of Sh, Sc and R are given by Cheng and Verma [2]. Equation (29) can be rewritten in dimensionless form as

$$\frac{Nu_x}{Ra_{v,x}^{1/2}} = \chi^{1/2}\frac{D^n(\chi)H(\chi)}{G(\chi)}[-\theta_v'(0)] \qquad (30)$$

where $Nu_x = q_w x/k_{m,v}(T_w - T_s)$ and $Ra_{v,x} = K(\rho_\infty - \rho_v)gx/\mu_v\alpha_v$ are the local Nusselt number and the local Rayleigh number of the vapor respectively. The average surface heat flux is given by

$$\bar{q} = (2/A)\int_0^{x_0}[\pi r(x)^n q_w(x)dx = (2l^n/A)k_{m,v}Ra_v^{1/2}C(T_w - T_s)[-\theta_v'(0)] \qquad (31)$$

where A is the area of the heating surface and

$$C = \pi^n\int_0^{\chi_0}\frac{D^{2n}(\chi)H(\chi)}{G(\chi)}d\chi$$

where $\chi_0 = x_0/l$ with x_0 denoting the total length of the surface in the x-direction.

We shall now obtain the explicit expressions for $G(\chi)$, $H(\chi)$, $D(\chi)$ and for the local and average Nusselt numbers for the special cases detailed below.

(i) *Wedge with half angle α and a slant height* L. For this case, we have $n = 0$, $\phi = \pi/2 - \alpha$, $H(\chi) = \cos\alpha$, $G(\chi) = (\chi\cos\alpha)^{1/2}$, $x_0 = l = L$, and $A = 2L$. Consequently, the local Nusselt number and the average Nusselt number are

$$Nu_x/Ra_{v,x}^{1/2} = -\theta_v'(0),$$
$$Nu_L/Ra_{v,L}^{1/2} = 2[-\theta_v'(0)] \qquad (32a, b)$$

where $Nu_L = \bar{q}L/k_{m,v}(T_w - T_s)$, $Ra_{v,x} = K(\rho_\infty - \rho_v)g\cos\alpha x/\mu_v\alpha_v$ and $Ra_{v,L} = K(\rho_\infty - \rho_v)g\cos\alpha L/\mu_v\alpha_v$ are the Rayleigh numbers based on the component of the gravitational force parallel to the inclined surface.

(ii) *Cone with half angle α and a slant height* L. For this case, we have $n = 1$, $\phi = \pi/2 - \alpha$, $x_0 = l = L$, $D(\chi) = \chi\sin\alpha$, $H(\chi) = \cos\alpha$, $G(\chi) = \chi\sin\alpha[\chi\cos\alpha/3]^{1/2}$ and $A = \pi L^2\sin\alpha$. Consequently

$$Nu_x/Ra_{v,x}^{1/2} = \sqrt{3}[-\theta_v'(0)] = 1.73[-\theta_v'(0)]$$
$$Nu_L/Ra_{v,L}^{1/2} = (4/\sqrt{3})[-\theta_v'(0)] = 2.30[-\theta_v'(0)] \qquad (33a, b)$$

where $Ra_{v,x}$ and Ra_v are defined as in Case (i). Note that Cases (i) and (ii) have been considered in refs. [2, 4].

(iii) *Horizontal cylinder with radius* r_0. For this case, we have $n = 0$, $l = r_0$, $\phi = x/r_0 = \chi$, $H(\chi) = \sin\chi$, $G(\chi) = (1 - \cos\chi)^{1/2}$, $x_0 = \pi r_0$, $\chi_0 = \pi$ and $A = 2\pi r_0$. Consequently,

$$Nu_x/(Ra_{v,x}^{1/2}) = (x/r_0)^{1/2}\frac{\sin(x/r_0)}{[1 - \cos(x/r_0)]^{1/2}}[-\theta_v'(0)], \qquad (34a)$$

$$Nu_d/(Ra_{v,d}^{1/2}) = (4/\pi)[-\theta_v'(0)] = 1.27[-\theta_v'(0)] \qquad (34b)$$

where $Nu_d = \bar{q}d/k_{m,v}(T_w - T_s)$ and $Ra_{v,d} = K(\rho_\infty - \rho_v)gd/\mu_v\alpha_v$ with $d = 2r_0$ denoting the diameter of the cylinder.

(iv) *Sphere with radius* r_0. For this case, we have $n = 1$, $l = r_0$, $\phi = x/r_0 = \chi$, $D(\chi) = \sin\chi$, $H(\chi) = \sin\chi$, $G(\chi) = [\cos^3\chi/3 - \cos\chi + 2/3]^{1/2}$, $x_0 = \pi r_0$, $\chi_0 = \pi$ and $A = 4\pi r_0^2$. Consequently,

$$\frac{Nu_x}{Ra_{v,x}^{1/2}} = (x/r_0)^{1/2}\frac{\sin^2(x/r_0)}{\left[\frac{\cos^3(x/r_0)}{3} - \cos(x/r_0) + 2/3\right]^{1/2}}[-\theta_v'(0)] \qquad (35a)$$

$$\frac{Nu_d}{Ra_{v,d}^{1/2}} = 2(2/3)^{1/2}[-\theta_v'(0)] = 1.63[-\theta_v'(0)] \qquad (35b)$$

where Nu_d and $Ra_{v,d}$ are defined in Case (iii). Note that in equations (34b) and (35b), it has been assumed that film boiling also exist at the top of the horizontal cylinder and the sphere.

REFERENCES

1. E. M. Parmentier, Two-phase natural convection adjacent to a vertical heated surface in a permeable medium, *Int. J. Heat Mass Transfer* **22**, 849–855 (1979).
2. P. Cheng and A. K. Verma, The effect of subcooled liquid on film boiling about a vertical heated surface in a porous medium, *Int. J. Heat Mass Transfer* **24**, 1151–1160 (1981).
3. L. P. Kwok and P. Cheng, Film boiling on the outer surface of a heated vertical cylinder embedded in a subcooled permeable medium, accepted for presentation, 7th International Heat Transfer Conference, September 6–10 1982, Munich, Germany.
4. P. Cheng, Film condensation along an inclined surface in a porous medium, *Int. J. Heat Mass Transfer* **24**, 983–990 (1981).
5. J. H. Merkin, Free convection boundary layers on axisymmetric and two-dimensional bodies of arbitrary shape in a saturated porous medium, *Int. J. Heat Mass Transfer* **22**, 1461–1462 (1979).